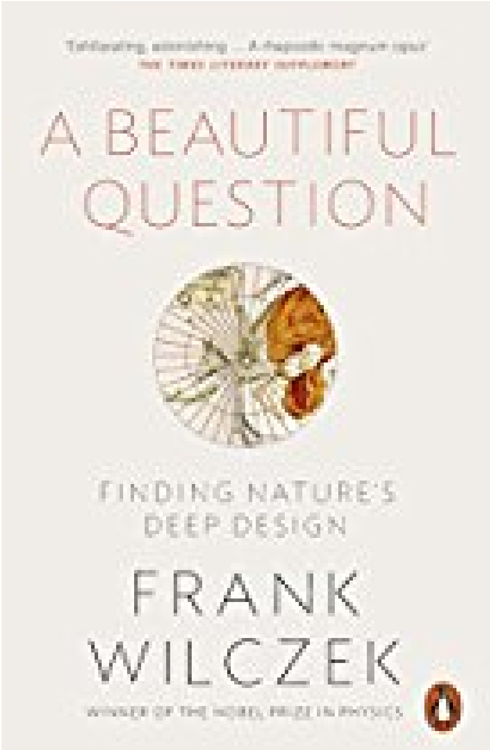

A Beautiful Question : Finding Nature's Deep Design

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448 pages
Extrait

This work was prepared especially for A Beautiful Question by He Shuifa,a modern master of traditional Chinese art and calligraphy. He is renownedfor the vigor and subtlety of his brushwork and for the spiritual depth of hisdepictions of flowers, birds, and nature. A simple translation of the inscriptionis this: “Taiji double fish is the essence of Chinese culture. This imagewas painted by He Shuifa on a lake in early winter.” The playful “doublefish” aspect of Taiji comes to life in He Shuifa’s image. The yin and yangresemble two carp playing together, and there are hints of their eyes and fins.In Henan, on the Yellow River, there is a waterfall called Dragon’s Gate.Yulong carp attempt to jump the cataract, although it is very difficult forthem. Those that succeed transform into lucky dragons. With a sense ofhumor, we may associate this event with the transformation of virtual intoreal particles, an essential quantum process that is now thought to underliethe origin of structure in the Universe (see plates XX and AAA). Alternativelywe may identify ourselves with the carp, and their strivings with ourquest for understanding.USER’S MANUAL

• The “Timelines” are mainly focused on events mentioned or alluded to in the book. They do what timelines do. They are not intended to be complete histories of anything, and they aren’t.

• The “Terms of Art” section contains explanatory definitions and discussions of key terms and concepts that occur in the main text. As you can infer from its length, it is rather more than a standard glossary. It contains alternative perspectives on many ideas in the text, and develops a few in new directions.

• The “Notes” section contains material that might, in an academic setting, have gone into footnotes. It both qualifies the text and provides some more technical references on particular points. You’ll also find a pair of poems in there.

• The brief “Recommended Reading” section is not a routine list of popularizations, nor of textbooks, but a carefully considered set of recommendations for further exploration in the spirit of the text, emphasizing primary sources.

I hope you’ve already enjoyed the cover art and the frontispiece, which set the tone for our meditation beautifully.There’s also a “User’s Manual”—but you knew that.THE QUESTIONThis book is a long meditation on a single question:Does the world embody beautiful ideas?Our Question may seem like a strange thing to ask. Ideas are one thing, physical bodies are quite another. What does it mean to “embody” an “idea”?Embodying ideas is what artists do. Starting from visionary conceptions, artists produce physical objects (or quasi-physical products, like musical scores that unfold into sound). Our Beautiful Question, then, is close to this one:Is the world a work of art?Posed this way, our Question leads us to others. If it makes sense to consider the world as a work of art, is it a successful work of art? Is the physical world, considered as a work of art, beautiful? For knowledge of the physical world we call on the work of scientists, but to do justice to our questions we must also bring in the insights and contributions of sympathetic artists.SPIRITUAL COSMOLOGYOur Question is a most natural one, in the context of spiritual cosmology. If an energetic and powerful Creator made the world, it could be that what moved Him—or Her, or Them, or It—to create was precisely an impulse to make something beautiful. Natural though it may be, this is assuredly *not* an orthodox idea, according to most religious traditions. Many motivations have been ascribed to the Creator, but artistic ambition is rarely prominent among them.In Abrahamic religions, conventional doctrine holds that the Creator set out to embody some combination of goodness and righteousness, and to create a monument to His glory. Animistic and polytheistic religions have envisaged beings and gods who create and govern different parts of the world with many kinds of motives, running the gamut from benevolence to lust to carefree exuberance.On a higher theological plane, the Creator’s motivations are sometimes said to be so awesome that finite human intellects can’t hope to comprehend them. Instead we are given partial revelations, which are to be believed, not analyzed. Or, alternatively, God is Love. None of those contradictory orthodoxies offers compelling reasons to expect that the world embodies beautiful ideas; nor do they suggest that we should strive to find such ideas. Beauty can form part of their cosmic story, but it is generally regarded as a side issue, not the heart of the matter.Yet many creative spirits have found inspiration in the idea that the Creator might be, among other things, an artist whose esthetic motivations we can appreciate and share—or even, in daring speculation, that the Creator is *primarily* a creative artist. Such spirits have engaged our Question, in varied and evolving forms, across many centuries. Thus inspired, they have produced deep philosophy, great science, compelling literature, and striking imagery. Some have produced works that combine several, or all, of those features. These works are a vein of gold running back through our civilization.Galileo Galilei made the beauty of the physical world central to his own deep faith, and recommended it to all:The greatness and the glory of God shine forth marvelously in all His works, and is to be read above all in the open book of the heavens.. . . as did Johannes Kepler, Isaac Newton, and James Clerk Maxwell. For all these searchers, finding beauty embodied in the physical world, reflecting God's glory, was the goal of their search. It inspired their work, and sanctified their curiosity. And with their discoveries, their faith was rewarded.While our Question finds support in spiritual cosmology, it can also stand on its own. And though its positive answer may inspire a spiritual interpretation, it does not require one.We will return to these thoughts toward the end of our meditation, by which point we will be much better prepared to appraise them. Between now and then, the world can speak for itself.HEROIC VENTURESJust as art has a history, with developing standards, so does the concept of the world as a work of art. In art history, we are accustomed to the idea that old styles are not simply obsolete, but can continue to be enjoyed on their own terms, and also offer important context for later developments. Though that idea is much less familiar in science, and in science it is subject to important limitations, the historical approach to our Question offers many advantages. It allows us—indeed, forces us—to proceed from simpler to more complex ideas. At the same time, by exploring how great thinkers struggled and often went astray, we gain perspective on the initial strangeness of ideas that have become, through familiarity, too "obvious" and comfortable. Last but by no means least, we humans are especially adapted to think in story and narrative, to associate ideas with names and faces, and to find tales of conflicts and their resolution compelling, even when they are conflicts of ideas, and no blood gets spilled. (Actually, a little does . . .)For these reasons we will sing, to begin, songs of heroes: Pythagoras, Plato, Filippo Brunelleschi, Newton, Maxwell. (Later a major heroine, Emmy Noether, will enter too.) Real people went by those names—very interesting ones! But for us they are not merely people, but also legends and symbols. I've portrayed them, as I think of them, in that style, emphasizing clarity and simplicity over scholarly nuance. Here biography is a means, not an end. Each hero advances our meditation several steps:

• *Pythagoras* discovered, in his famous theorem about right-angled triangles, a most fundamental relationship between numbers, on the one hand, and sizes and shapes, on the other. Because Number is the purest product of Mind, while Size is a primary characteristic of Matter, that discovery revealed a hidden unity between Mind and Matter.

Pythagoras also discovered, in the laws of stringed instruments, simple and surprising relationships between numbers and musical harmony. That discovery completes a trinity, Mind-Matter-Beauty, with Number as the linking thread. Heady stuff! It led Pythagoras to surmise that All Things Are Number. With these discoveries and speculations, our Question comes to life.

• *Plato* thought big. He proposed a geometric theory of atoms and the Universe, based on five symmetrical shapes, which we now call the Platonic solids. In this audacious model of physical reality, Plato valued beauty over accuracy. The details of his theory are hopelessly wrong. Yet it provided such a dazzling vision of what a positive answer to our Question might look like that it inspired Euclid, Kepler, and many others to brilliant work centuries later. Indeed, our modern, astoundingly successful theories of elementary particles, codified in our Core Theory (see page 8), are rooted in heightened ideas of symmetry that would surely make Plato smile. And when trying to guess what will come next, I often follow Plato's strategy, proposing objects of mathematical beauty as models for Nature.

Plato was also a great literary artist. His metaphor of the Cave captures important emotional and philosophical aspects of our relationship, as human inquirers, with reality. At its core is the belief that everyday life offers us a mere shadow of reality, but that through adventures of mind, and sensory expansion, we can get to its essence—and that the essence is clearer and more beautiful than its shadow. He imagined a mediating *demiurge,* which can be translated as *Artisan,* who rendered the realm of perfect, eternal Ideas into its imperfect copy, the world we experience. Here the concept of the world as a work of art is explicit.

- *Brunelleschi* brought new ideas to geometry from the needs of art and engineering. His *projective geometry,* in dealing with the actual appearance of things, brought in ideas—relativity, invariance, symmetry—not only beautiful in themselves, but pregnant with potential.
- *Newton* brought the mathematical understanding of Nature to entirely new levels of ambition and precision.

A common theme pervades Newton's titanic work on light, the mathematics of calculus, motion, and mechanics. It is the method he called Analysis and Synthesis. The method of Analysis and Synthesis suggests a two-stage strategy to achieve understanding. In the analysis stage, we consider the smallest parts of what we are studying their "atoms," using the word figuratively. In a successful analysis, we identify small parts that have simple properties that we can summarize in precise laws. For example:

- In the study of light, the atoms are beams of pure spectral colors.
- In the study of calculus, the atoms are infinitesimals and their ratios.
- In the study of motion, the atoms are velocity and acceleration.
- In the study of mechanics, the atoms are forces.

(We'll discuss these in more depth later.) In the synthesis stage we build up, by logical and mathematical reasoning, from the behavior of individual atoms to the description of systems that contain many atoms.When thus stated broadly, Analysis and Synthesis may not seem terribly impressive. It is, after all, closely related to common rules of thumb, e.g., "to solve a complex problem, divide and conquer"—hardly an electrifying revelation. But Newton demanded precision and completeness of understanding, saying,'Tis much better to do a little with certainty & leave the rest for others that come after than to explain all things by conjecture without making sure of any thing.And in these impressive examples, he achieved his ambitions. Newton showed, convincingly, that Nature herself proceeds by Analysis and Synthesis. There really is simplicity in the "atoms," and Nature really does operate by letting them do their thing.Newton also, in his work on motion and mechanics, enriched our concept of what physical laws are. His laws of motion and of gravity are *dynamical* laws. In other words, they are laws of change. Laws of this kind embody a different concept of beauty than the static perfection beloved of Pythagoras and (especially) Plato.Dynamical beauty transcends specific objects and phenomena, and invites us to imagine the expanse of possibilities. For example, the sizes and shapes of actual planetary orbits are not simple. They are neither the (compounded) circles of Aristotle, Ptolemy, and Nicolaus Copernicus, nor even the more nearly accurate ellipses of Kepler, but rather curves that must be calculated numerically, as functions of time, evolving in complicated ways that depend on the positions and masses of the Sun and the other planets. There is great beauty and simplicity here, but it is only fully evident when we understand the deep design. The appearance of particular objects does not exhaust the beauty of the laws.

- *Maxwell* was the first truly modern physicist. His work on electromagnetism ushered in both a new concept of reality and a new method in physics. The new concept, which Maxwell developed from the intuitions of Michael Faraday, is that the primary ingredients of physical reality are not point-like *particles,* but rather space-filling *fields.* The new method is *inspired guesswork.* In 1864 Maxwell codified the known laws of electricity and magnetism into a system of equations, but discovered the resulting system was inconsistent. Like Plato, who shoehorned five perfect solids into four elements plus the Universe, Maxwell did not give up. He saw that by adding a new term he could both make the equations appear more symmetric and make them mathematically consistent. The resulting system, known as the Maxwell equations, not only unified electricity and magnetism, but derived light as a consequence, and survives to this day as the secure foundation of those subjects.

By what is the physicist's "inspired guesswork" inspired? Logical consistency is necessary, but hardly sufficient. Rather it was beauty and symmetry that guided Maxwell and his followers—that is, all modern physicists—closer to truth, as we shall see.Maxwell also, in his work on color perception, discovered that

Plato's metaphorical Cave reflects something quite real and specific: the paltriness of our sensory experience, relative to available reality. And his work, by clarifying the limits of perception, allows us to transcend those limits. For the ultimate sense-enhancing device is a searching mind.QUANTUM FULFILLMENTThe definitive answer "yes" to our Question came only in the twentieth century, with the development of quantum theory.The quantum revolution gave this revelation: we've finally learned what Matter is. The necessary equations are part of the theoretical structure often called the Standard Model. That yawn-inducing name hardly does the achievement justice, and I'm going to continue my campaign, begun in *The Lightness of Being*, to replace it with something more appropriately awesome:Standard Model Core TheoryThis change is more than justified, because1. "Model" connotes a disposable makeshift, awaiting replacement by the "real thing." But the Core Theory is already an accurate representation of physical reality, which any future, hypothetical "real thing" must take into account.2. "Standard" connotes "conventional," and hints at superior wisdom. But no such superior wisdom is available. In fact, I think—and mountains of evidence attest—that while the Core Theory will be supplemented, its core will persist.The Core Theory embodies beautiful ideas. The equations for atoms and light are, almost literally, the same equations that govern musical instruments and sound. A handful of elegant designs support Nature's exuberant construction, from simple building blocks, of the material world.Our Core Theories of the four forces of Nature—gravity, electromagnetism, and the strong and weak forces—embody, at their heart, a common principle: *local symmetry.* As you will read, this principle both fulfills and transcends the yearnings of Pythagoras and Plato for harmony and conceptual purity. As you will *see,* this principle both builds upon and transcends the artistic geometry of Brunelleschi and the brilliant insights of Newton and Maxwell into the nature of color.The Core Theory completes, for practical purposes, the analysis of matter. Using it, we can *deduce* what sorts of atomic nuclei, atoms, molecules—and stars—exist. And we can reliably orchestrate the behavior of larger assemblies of these elements, to make transistors, lasers, or Large Hadron Colliders. The equations of the Core Theory have been tested with far greater accuracy, and under far more extreme conditions, than are required for applications in chemistry, biology, engineering, or astrophysics. While there certainly are many things we don't understand—I'll mention some important ones momentarily!—we do understand the Matter we're made from and that we encounter in normal life (even if we're chemists, engineers, or astrophysicists).Despite its overwhelming virtues, the Core Theory is imperfect. Indeed, precisely because it is such a faithful description of reality, we must, in pursuit of our Question, hold it to the highest esthetic standards. So scrutinized, the Core Theory reveals flaws. Its equations are lopsided, and they contain several loosely connected pieces. Furthermore, the Core Theory does not account for so-called dark matter and dark energy. Although those tenuous forms of matter are negligible in our immediate neighborhood, they persist in the interstellar and intergalactic voids, and thereby come to dominate the overall mass of the Universe. For those and other reasons, we cannot remain satisfied.Having tasted beauty at the heart of the world, we hunger for more. In this quest there is, I think, no more promising guide than beauty itself. I shall show you some hints that suggest concrete possibilities for improving our description of Nature. As I aspire to inspired guesswork, beauty is my inspiration. Several times it's worked well for me, as you'll see.VARIETIES OF BEAUTYDifferent artists have different styles. We don't expect to find Renoir's shimmering color in Rembrandt's mystic shadows, or the elegance of Raphael in either. Mozart's music comes from a different world entirely, the Beatles' from another, and Louis Armstrong's from yet another. Likewise, the beauty embodied in the physical world is a particular kind of beauty. Nature, as an artist, has a distinctive style.To appreciate Nature's art, we must enter her style with sympathy. Galileo, ever eloquent, expressed it this way:Philosophy [Nature] is written in that great book which ever is before our eyes—I mean the universe—but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles, and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.Today we've penetrated much further into the great book, and discovered that its later chapters use a more imaginative, less familiar language than the Euclidean geometry Galileo knew. To become a fluent speaker in it is the work of a lifetime (or at least of several years in graduate school). But just as a graduate degree in art history is not a prerequisite for engaging with the world's best art and finding that a deeply rewarding experience, so I hope, in this book, to help you engage with Nature's art, by making her style accessible. Your effort will be rewarded, for as Einstein might have said,Subtle is the Lord, but malicious She is not.Two obsessions are the hallmarks of Nature's artistic style:

- Symmetry—a love of harmony, balance, and proportion
- Economy—satisfaction in producing an abundance of effects from very limited means

Watch for these themes as they recur, grow, and develop throughout our narrative and give it unity. Our appreciation of them has evolved from intuition and wishful thinking into precise, powerful, and fruitful methods.Now, a disclaimer. Many varieties of beauty are underrepresented in Nature's style, as expressed in her fundamental operating system. Our delight in the human body and our interest in expressive portraits, our love of animals and of natural landscapes, and many other sources of artistic beauty are not brought into play. Science isn't everything, thank goodness.CONCEPTS AND REALITIES; MIND AND MATTEROur Question can be read in two directions. Most obviously, it is a question about the world. That is the direction we've emphasized so far. But the other direction is likewise fascinating. When we find that *our* sense of beauty is realized in the physical world, we are discovering something about the world, but also something about ourselves.Human appreciation of the fundamental laws of Nature is a recent development on evolutionary or even historical time scales. Moreover, those laws reveal themselves only after elaborate operations—looking through sophisticated microscopes and telescopes, tearing atoms and nuclei apart, and processing long chains of mathematical reasoning—that do not come naturally. Our sense of beauty is not in any very direct way adapted to Nature's fundamental workings. Yet just as surely, our sense of beauty is excited by what we find there.What explains that miraculous harmony of Mind and Matter? Without an explanation of that miracle, our Question remains mysterious. It is an issue our meditation will touch upon repeatedly. For now, two brief anticipations:1. We human beings are, above all, visual creatures. Our sense of vision, of course, and in a host of less obvious ways our deepest modes of thought, are conditioned by our interaction with light. Each of us, for example, is born to become an accomplished, if unconscious, practitioner of projective geometry. That ability is hardwired into our brain. It is what allows us to interpret the two-dimensional image that arrives on our retinas as representing a world of objects in three-dimensional space.Our brains contain specialized modules that allow us to construct, very quickly and without conscious effort, a dynamic worldview based on three-dimensional objects located in three-dimensional space. We do this beginning from two two-dimensional images on the retinas of our eyes (which, in turn, are the product of light rays emitted or reflected from the surfaces of external objects, which propagate to us in straight lines). To work back from the images we receive to the objects that cause them is a tricky problem in inverse projective geometry. In fact, as stated, it is an impossible problem, because there's not nearly enough information in the projections to do an unambiguous reconstruction. A basic problem is that even to get started we need to separate objects from their background (or foreground). We exploit all kinds of tricks based on typical properties of objects we encounter, such as their color or texture contrast and distinctive boundaries, to do that job. But even after that step is accomplished, we are left with a difficult geometrical problem, for which Nature has helpfully provided us, in our visual cortex, an excellent specialized processor.Another important feature of vision is that light arrives to us from very far away, and gives us a window into astronomy. The regular apparent motion of stars and the slightly less regular apparent motion of planets gave early hints of a lawful Universe, and provided an early inspiration and testing ground for the mathematical description of Nature. Like a good textbook, it contains problems with varying degrees of difficulty.In the more advanced, modern parts of physics we learn that light itself is a form of matter, and indeed that matter in general, when understood deeply, is remarkably light-like. So again, our interest in and experience with light, which is deeply rooted in our essential nature, proves fortunate.Creatures that, like most mammals, perceive the world primarily through the sense of smell would have a much harder time getting to physics as we know it, even if they were highly intelligent in other ways. One can imagine dogs, say, evolving into extremely intelligent social creatures, developing language, and experiencing rich lives full of interest and joy, but devoid of the specific kinds of curiosity and outlook, based on visual experience, that lead to our kind of deep understanding of the physical world. Their world would be rich in reactions and decays—they'd have great chemistry sets, elaborate cuisines, aphrodisiacs, and, à la Proust, echoing memories. Projective geometry and astronomy, maybe not so much. We understand that smell is a chemical sense, and we are beginning to understand its foundation in molecular events. But the "inverse" problem of working from smell back to molecules and their laws, and eventually to physics as we know it, seems to me hopelessly difficult.Birds, on the other hand, are visual creatures, like us. Beyond that, their way of life would give them an extra advantage over humans, in getting started on physics. For birds, with their freedom of flight, experience the essential symmetry of three-dimensional space in an intimate way that we do not. They also experience the basic regularities of motion, and especially the role of inertia, in their everyday lives, as they operate in a nearly frictionless environment. Birds are born, one might say, with intuitive knowledge of classical mechanics and Galilean relativity, as well as of geometry. If some species of bird evolved high abstract intelligence—that is, if they ceased being birdbrains—their physics would develop rapidly. Humans, on the other hand, have to unlearn the friction-laden Aristotelean mechanics they use in everyday life, in order to achieve deeper understanding. Historically that involved quite a struggle!Dolphins, in their watery environment, and bats, with their echolocation, give us other interesting variations on these themes. But I will not develop those here.A general philosophical point, which these considerations illustrate, is that the world does not provide its own unique interpretation. The world offers many possibilities for different sensory universes, which support very different interpretations of the world's significance. In this way our so-called Universe is already very much a multiverse.2. Successful perception involves sophisticated inference, because the information we sample about the world is both very partial and very noisy. For all our innate powers, we must also learn how to see by interacting with the world, forming expectations, and comparing our predictions with reality. When we form expectations that turn out to be correct, we experience pleasure and satisfaction. Those reward mechanisms encourage successful learning. They also stimulate—indeed, at base they *are*—our sense of beauty.Putting those observations together, we discover an explanation of why we find interesting phenomena (phenomena we can learn from!) in physics beautiful. An important consequence is that we especially value experience that is surprising, but not too surprising. Routine, superficial recognition will not challenge us, and may not be rewarded as active learning. On the other hand, patterns whose meaning we cannot make sense of at all will not offer rewarding experience either; they are noise.And here we are lucky too, in that Nature employs, in her basic workings, symmetry and economy of means. For these principles, like our intuitive understanding of light, promote successful prediction and learning. From the appearance of part of a symmetric object we can predict (successfully!) the appearance of the rest; from the behavior of parts of natural objects we can predict (sometimes successfully!) the behavior of wholes. Symmetry and economy of means, therefore, are exactly the sorts of things we are apt to experience as beautiful.NEW IDEAS AND INTERPRETATIONSTogether with new appreciations of some very old and some less old ideas, you will find in this book several essentially new ones. Here I'd like to mention some of the most important.My presentation of the Core Theory as geometry, and my speculations about the next steps beyond it, are adaptations of my technical work in fundamental physics. That work builds, of course, on the work of many others. My use of color fields as an example of extra dimensions, and my exploitation of the possibilities they open up for illustrating local symmetry, are (as far as I know) new.My theory that promotion of learning underlies, and is the evolutionary cause of, our sense of beauty in important cases, and the application of that theory to musical harmony, which offers a rational explanation for Pythagoras's discoveries in music, form a constellation of ideas I've entertained privately for a long time but present here for the first time publicly. Caveat emptor.My discussion of the expansion of color perception draws on an ongoing program of practical research that I hope will lead to commercial products. Patents have been applied for.I'd like to think that Niels Bohr would approve of my broad interpretation of complementarity, and might even acknowledge his paternity—but I'm not sure he would.PYTHAGORAS I: THOUGHT AND OBJECTTHE SHADOW PYTHAGORASThere was a person named Pythagoras who lived and died around 570–495 BCE, but very little is known about him. Or rather a lot is "known" about him, but most of it is surely wrong, because the documentary trail is littered with contradictions. It combines the sublime, the ridiculous, the unbelievable, and the just plain weird.Pythagoras was said to be the son of Apollo, to have a golden thigh, and to glow. He may or may not have advocated vegetarianism. Among his most notorious sayings is an injunction not to eat beans, because "beans have a soul." Yet several early sources explicitly deny that Pythagoras said or believed anything of the sort. More reliably, Pythagoras believed in, and taught, the transmigration of souls. There are several stories—each, to be sure, dubious—that corroborate this. According to Aulus Gellius, Pythagoras remembered four of his own past lives, including one as a beautiful courtesan named Alco. Xenophanes recounts that Pythagoras, upon hearing the cries of a dog who was being beaten, rushed to halt the beating, claiming to recognize the voice of a departed friend. Pythagoras also, like Saint Francis centuries later, preached to animals.The *Stanford Encyclopedia of Philosophy*—a free and extremely valuable online resource, by the way—sums it up as follows:The popular modern image of Pythagoras is that of a master mathematician and scientist. The early evidence shows, however, that, while Pythagoras was famous in his own day and even 150 years later in the time of Plato and Aristotle, it was not mathematics or science upon which his fame rested. Pythagoras was famous1. As an expert on the fate of the soul after death, who thought that the soul was immortal and went through a series of reincarnations2. As an expert on religious ritual3. As a wonder-worker who had a thigh of gold and who could be two places at the same time4. As the founder of a strict way of life that emphasized dietary restrictions, religious ritual and rigorous self disciplineA few things do seem clear. The historical Pythagoras was born on the Greek island of Samos, traveled widely, and became the inspiration for and founder of an unusual religious movement. His cult flourished briefly in Crotone, in southern Italy, and developed chapters in several other places before being everywhere suppressed. The Pythagoreans formed secretive societies, on which the initiates' lives centered. These communities, which included both men and women, promoted a kind of intellectual mysticism that seemed marvelous, yet strange and threatening, to most of their contemporaries. Their worldview centered on worshipful admiration of numbers and musical harmony, which they saw as reflecting the deep structure of reality. (As we'll see, they were on to something.)THE REAL PYTHAGORASHere again is the *Stanford Encyclopedia:*The picture of Pythagoras that emerges from the evidence is thus not of a mathematician, who offered rigorous proofs, or of a scientist, who carried out experiments to discover the nature of the natural world, but rather of someone who sees special significance in and assigns special prominence to mathematical relationships that were in general circulation.Bertrand Russell was pithier:A combination of Einstein and Mary Baker Eddy.To scholars of factual biography, it is a major problem that later followers of Pythagoras ascribed their own ideas and discoveries to Pythagoras himself. In that way they hoped both to give their ideas authority and, by enhancing Pythagoras's reputation, to promote their community—the community he founded. Thus magnificent discoveries in different fields of mathematics, physics, and music, as well as an inspiring mysticism, a seminal philosophy, and a pure morality were all portrayed as the legacy of a single godlike figure. That awesome figure is, for us, the *real* Pythagoras.It is not altogether inappropriate to assign the (historical) shadow Pythagoras credit for the real Pythagoras, because the latter's great achievements in mathematics and science emerged from the way of life the former inspired, and the community he founded.(Those so inclined might draw parallels to the differing careers in life, and afterward, of other major religious figures . . .)Thanks to Raphael, we know what the real Pythagoras looked like. In plate B* he is captured deep in concentration as he writes in a great book, surrounded by admirers.ALL THINGS ARE NUMBERIt is difficult to make out what Pythagoras is writing, but I like to pretend it is some version of his most fundamental credo:All Things Are NumberIt is also difficult to know, at this separation in time and space, exactly what Pythagoras meant by that. So we get to use our imagination.PYTHAGORAS'S THEOREMFor one thing, Pythagoras was mightily impressed by Pythagoras's theorem. So much so that when he discovered it, in a notable lapse from vegetarianism, he offered a hecatomb—the ritual sacrifice of one hundred oxen, followed by feasting—to the Muses, in thanks.Why the fuss?Pythagoras's theorem is a statement about right triangles; that is, triangles that contain a 90-degree angle, or, in other words, a square corner. The theorem tells you that if you erect squares on the different sides of such a triangle, then the sum of the areas of the two smaller squares adds up to the area of the largest square. A classic example is the 3-4-5 right triangle, shown in figure 1:FIGURE 1. THE 3-4-5 RIGHT TRIANGLE, A SIMPLE CASE OF PYTHAGORAS'S THEOREM.The areas of the two smaller squares are 32 = 9 and 42 = 16, as we can see, in the spirit of Pythagoras, by *counting* their subunits. The area of the largest square is 52 = 25. And we verify 9 + 16 = 25.By now Pythagoras's theorem is familiar to most of us, if only as a dim memory from school geometry. But if you listen to its message afresh, with Pythagoras's ears, so to speak, you realize that it is saying something quite startling. It is telling you that the *geometry* of objects embodies hidden *numerical* relationships. It says, in other words, that Number describes, if not yet everything, at least something very important about physical reality, namely the sizes and shapes of the objects that inhabit it.Later in this meditation we will be dealing with much more advanced and sophisticated concepts, and I'll have to resort to metaphors and analogies to convey their meaning. The special joy one finds in precise mathematical thinking, when sharply defined concepts fit together perfectly, is lost in translation. Here we have an opportunity to experience that special joy. Part of the magic of Pythagoras's theorem is that one can prove it with minimal preparation. The best proofs are unforgettable, and their memory lasts a lifetime. They've inspired Aldous Huxley and Albert Einstein—not to mention Pythagoras!—and I hope they'll inspire you.Guido's Proof"So simple!"That is what Guido, the young hero of Aldous Huxley's short story "Young Archimedes," says, as he describes his demonstration of Pythagoras's theorem. Guido's proof is based on the shapes displayed in plate C.Guido's PlaythingLet's spell out what was obvious to Guido at a glance.Each of the two large tiled squares contains four colored triangles that are matched in the other large square. All the colored triangles are right triangles, and all are the same size. Let's say the length of the smallest side is *a,* the next smallest *b,* and the longest (the hypotenuse) *c.* Then it's easy to see that the sides of both large (total) squares have length *a* + *b,* and in particular that those two squares have equal areas. So the non-triangular parts of the large squares must also have equal areas.But what are those equal areas? In the first large square, on the left, we have a blue square with side *a,* and a red square with side *b.* They have areas *a*2 and *b*2, and their combined area is *a*2 + *b*2. In the second large square, on the right, we have a gray square with side *c.* Its area is *c*2. Recalling the preceding paragraph, we conclude thata2 + b2 = c2 . . . which is Pythagoras's theorem!Einstein's Proof(?)In Einstein's *Autobiographical Notes* he recalls,I remember that an uncle told me about the Pythagorean theorem before the holy geometry booklet had come into my hands. After much effort I succeeded in "proving" this theorem on the basis of the similarity of triangles; in doing so it seemed to me "evident" that the relations of the sides of the right-angled triangles would have to be determined by one of the acute angles.There is not really enough detail in that account to reconstruct Einstein's demonstration with certainty, but here, in figure 2, is my best guess. That guess deserves to be right, because this is the simplest and most beautiful proof of Pythagoras's theorem. In particular, this proof makes it brilliantly clear why the *squares* of the lengths are what's involved in the theorem.FIGURE 2. A PLAUSIBLE RECONSTRUCTION OF EINSTEIN'S PROOF, FROM AUTOBIOGRAPHICAL NOTES.A Polished JewelWe start from the observation that right triangles that include a common angle ϕ are all similar to one another, in the precise sense that you can get from any one to any other by an overall rescaling (magnification or shrinking). Also: if we rescale the length of the triangle by some factor, then we will rescale the area by the square of that factor.Now consider the three right triangles that appear in figure 2: the total figure, and the two sub-triangles it contains. Each of them contains the angle ϕ, so they are similar. Their areas are therefore proportional to *a*2, *b*2, *c*2, going from smallest to largest. But because the two sub-triangles add up to the total triangle, the corresponding areas must also add up, and thereforea2 + b2 = c2 . . . Pythagoras's theorem pops right out!A Beautiful IronyIt is a beautiful irony that Pythagoras's theorem can be used to undermine his doctrine All Things Are Number.That scandalous result is the one discovery of the Pythagorean school that was not attributed to Pythagoras, but rather to his pupil Hippasus. Shortly after his discovery, Hippasus drowned at sea. Whether his death should be attributed to the wrath of the gods, or the wrath of the Pythagoreans, is a debated point.Hippasus's reasoning is very clever, but not overly complicated. Let's waltz through it.We consider isosceles right triangles with two equal sides—in other words, *a* = *b*. Pythagoras's theorem tells us that2 × a2 = c2Now let's suppose that the lengths *a* and *c* are both whole numbers. If *all* things are numbers, they'd better be! But we'll find that it's impossible.If both *a* and *c* are even numbers, we can consider a similar triangle of half the size. We can keep halving until we reach a triangle where at least one of *a*, *c* is odd.But whichever choice we make, we quickly derive a contradiction.First let's suppose that *c* is odd. Then so is c^2. But $2 \times a^2$ is obviously even because it contains a factor 2. So we can't have $2 \times a^2 = c^2$, as Pythagoras's theorem tells us. Contradiction!Alternatively, suppose that *c* is even, say $c = 2 \times p$. Then $c^2 = 4 \times p^2$. Then Pythagoras's theorem tells us, after we divide both sides by 2, that $a^2 = 2 \times p^2$. And so *a* can't be odd, by the same reasoning as before. Contradiction!So all things can't be whole numbers, after all. There cannot be an atom of length, such that all possible lengths are whole number multiples of that atom's length.It doesn't seem to have occurred to the Pythagoreans that one might draw a different conclusion, saving All Things Are Number. After all, one *can* imagine a world where space is constructed from many identical atoms. Indeed, my friends Ed Fredkin and Stephen Wolfram advocate models of our world based on cellular automata, which have exactly this property. And your computer screen, based on atoms of light we call pixels, shows that such a world can look pretty realistic! Logically, the correct conclusion to draw is that in such a world, one cannot construct exact isosceles right triangles. Something has to go slightly wrong. The "right" angle might fail to be exactly 90 degrees, or the two shorter sides might fail to be perfectly equal or—as on the computer screen—the sides of your triangles might fail to be exactly straight.This is not the option Greek mathematicians chose. Rather, they considered geometry in its more appealing continuous form, where we allow exact right angles and exact equality of sides to coexist. (This is also the choice that has proved most fruitful for physics, as we'll learn from Newton.) To do this, they had to prioritize geometry over arithmetic, because—as we've seen—the whole numbers are inadequate to describe even very simple geometric figures. Thus they abandoned the letter, though not the spirit, of All Things Are Number.THOUGHT AND OBJECTFor the true essence of Pythagoras's credo is not a literal assertion that the world must embody whole numbers, but the optimistic conviction that the world should embody *beautiful concepts.*The lesson for which Hippasus paid with his life is that we must be willing to learn from Nature what those concepts are. In this enterprise, humility is mandatory. Geometry is not less beautiful than arithmetic. Indeed, it is more naturally suited to our highly visual brains, and most people prefer it. And geometry is no less conceptual, no less a pure world of Mind, than arithmetic. Much of ancient Greek mathematics, epitomized in Euclid's *Elements,* was devoted to showing precisely this: that geometry is a system of *logic.*As we continue our meditation, we'll find that Nature is inventive in her language. She stretches our imagination with new kinds of numbers, new kinds of geometry—and even, in the quantum world, new kinds of logic.PYTHAGORAS II: NUMBER AND HARMONYThe essence of all stringed instruments, whether ancient lyre or modern guitar, cello, or piano, is the same: they produce sound from the motion of strings. The exact quality of sound, or timbre, depends on many complex factors, including the nature of the material that makes the string, the shapes of the surfaces—"sounding boards"—that vibrate in sympathy, and the way in which the string is plucked, bowed, or hammered. But in all instruments there is a principal tone, or pitch, that we recognize as the note being played. Pythagoras—the real one—discovered that the pitch obeys two remarkable rules. Those rules make direct connections among numbers, properties of the physical world, and our sense of harmony (which is one face of beauty).The drawing that follows, not by Raphael, shows Pythagoras in action, performing experiments on harmony:FIGURE 3. AN ETCHING FROM MEDIEVAL EUROPE DEPICTING PYTHAGORAS AT WORK ON MUSICAL HARMONY. WE CAN INFER FROM THE FIGURE THAT PYTHAGORAS LISTENED TO HOW THE SOUNDS PRODUCED BY HIS INSTRUMENT CHANGED AS HE VARIED TWO DIFFERENT THINGS. BY HOLDING A STRING DOWN FIRMLY AT DIFFERENT POINTS, HE COULD VARY THE EFFECTIVE LENGTH OF THE VIBRATING PART. AND BY CHANGING THE WEIGHT THAT STRETCHES A STRING, HE COULD VARY ITS TENSION.HARMONY, NUMBER, AND LENGTH: AN ASTONISHING CONNECTIONPythagoras's first rule is a relationship between the length of the vibrating string and our perception of its tone. The rule says that two copies of the same type of string, both subject to the same tension, make tones that sound good together precisely when the lengths of the strings are in ratios of small whole numbers. Thus, for example, when the ratio of lengths is 1:2, the tones form an octave. When the ratio is 2:3, we hear the dominant fifth; when the ratio is 3:4, the major fourth. In musical notation (in the key of C) these correspond to playing two Cs, one above the other, together, a C-G, or C-F, respectively. People find those tone combinations appealing. They are the main building blocks of classical music, and of most folk, pop, and rock music.In applying Pythagoras's rule, the length that we must consider is of course the effective length, that is, the length of the portion of the string that actually vibrates. By clamping down on the string, creating a dead zone, we can change the tone. Guitarists and cellists exploit that possibility when they "finger" with their left hands. As they do so they are, whether or not they know it, reincarnating Pythagoras. In the drawing, we see Pythagoras adjusting the effective length using a pointed clamp, which is a technique conducive to accurate measurement.When tones sound good together, we say they are in harmony, or that they are concordant. What Pythagoras discovered, then, is that the perceived harmonies of tones reflect relationships in what might seem to be an entirely different world—the world of numbers.HARMONY, NUMBER, AND WEIGHT: AN ASTOUNDING CONNECTIONPythagoras's second rule involves the tension of the string. The tension can be adjusted, in a controlled and readily measurable way, by burdening the string with different amounts of weight, as shown in figure 3. Here the result is even more remarkable. The tones are in harmony if the tensions are ratios of *squares* of small whole numbers. Higher tensions correspond to higher pitches. Thus a 1:4 ratio of tensions produces the octave, and so forth. When string musicians tune their instruments prior to a performance, stretching or relaxing the strings by winding their pegs, Pythagoras returns.This second relationship is even more impressive than the first as evidence that Things are hidden Numbers. The relationship is better hidden because the numbers must be processed—squared, to be exact—before the relationship becomes evident. The shock of discovery is accordingly greater. Also, the relationship brings in weight. And weight, more unmistakably than length, links us to Things in the material world.DISCOVERY AND WORLDVIEWNow we've discussed three major Pythagorean discoveries: the Pythagorean theorem on right triangles, and two rules of musical consonance. Together, they link shape, size, weight, and harmony, with the common thread being Number.For the Pythagoreans, that trinity of discoveries was more than enough to anchor a mystic worldview. Vibration of strings is the source of musical sound. These vibrations are nothing but periodic motions; that is, motions which repeat themselves at regular intervals. We also see the Sun and planets move in periodic motions across the sky, and infer their periodic motion in space. So they too must emit sound. Their sounds form the Music of the Spheres, a music that fills the cosmos.Pythagoras was fond of singing. He also claimed actually to hear the Music of the Spheres. Some modern scholars speculate that the historical Pythagoras suffered from tinnitus, or ringing in the ears. The real Pythagoras, of course, did not.In any case, the larger point is that All Is Number, and Number supports Harmony. The Pythagoreans, drunk on mathematics, inhabited a harmony-filled world.THE FREQUENCY IS THE MESSAGEPythagoras's musical rules deserve, I think, to be considered the first quantitative laws of Nature ever discovered. (Astronomical regularities, beginning with the regular alternation of night and day, were of course noticed much earlier. Calendar-keeping and casting of horoscopes, using mathematics to predict or reconstruct the positions of the Sun, Moon, and planets, were significant technologies before Pythagoras was born. But empirical observations about specific objects are quite different from general laws of Nature.)It is ironic, therefore, that we still don't fully understand why they are true. Today we have a much better understanding of the physical processes involved in the production, transmission, and reception of sound, but the connection between that knowledge and the perception of "notes that sound good together" has so far been elusive. I think there is a promising set of ideas about that. These ideas are close to the central concern of our meditation, because (if true) they elucidate an important origin of our sense of beauty.Our account of the *why* of Pythagoras's rules has three parts. The first part starts with the vibrating string and proceeds to our eardrums. The second part starts with the eardrum and proceeds to primary nerve impulses. The third part starts with primary nerve pulses and proceeds to perceived harmony.The vibration of a string goes through several transformations before arriving to our minds as a message. The vibration disturbs the surrounding air directly, simply by pushing it. The hum of an isolated string is quite weak, however. Practical musical instruments employ sounding boards, which respond to the string's vibration with stronger vibrations of their own. The motion of the sounding board pushes air around more robustly.The disturbance of air near the string or sounding board then takes on a life of its own, becoming a propagating disturbance: a sound wave that spreads outward in all directions. Any sound wave is a recurring cycle of compression and decompression. The vibrating air in each region of space exerts pressure on neighboring regions and sets them into vibration. Eventually a portion of this sound wave, funneled by the complicated geometry of the ear, arrives at a membrane called the eardrum a few centimeters within. Our eardrums serve as inverse sounding boards, where now vibrations of air induce mechanical motion, instead of the opposite.The eardrum vibrations set off more reactions, as we'll discuss momentarily. Before that, however, we should make a simple but fundamental observation. This long series of transformations can seem bewildering, and one may wonder how a meaningful signal, reflecting what that string was doing, can be extracted far down the line. The point is that throughout all these transformations there is a property that remains unchanged. The rate of the vibrations in time or, as we say, their *frequency*, whether they are vibrations in string, in sounding board, in air, or in eardrum—or in the ossicles, cochlear fluid, basilar membrane, and hair cells farther down the line—remains the same. For at each transformation, the pushes and pulls of one stage induce the compressions and decompressions of the next, one for one, and so the different kinds of disturbances are synchronized or, as we say, "in time." We can anticipate, therefore, and will find, that the useful things to monitor, if we want our perception to reflect a property of the initial vibration, is the frequency of vibrations it eventually sets up in our heads.The first step toward understanding Pythagoras's rules, therefore, is to cast them in terms of frequency. Today we have reliable equations of mechanics that allow us to calculate how the frequency of vibration of a string changes as we vary its length or tension. Using those equations, we find that the frequency falls proportionally to the length, and rises proportionally to the square root of the tension. Therefore Pythagoras's rules, translated into frequency, both make the same simple statement. They both state that notes sound good together if their frequencies are in ratios of small whole numbers.A THEORY OF HARMONYNow let us resume our story, at its second stage. The eardrum is attached to a system of three small bones, the ossicles, which in turn are attached to a membranous "oval window" opening on a snail-like structure, the cochlea. The cochlea is the critical organ for hearing, playing a role roughly analogous to the role the eye plays in sight. It is filled with fluid that is set in motion by the vibrations at the oval window. Immersed in that fluid is a long tapering membrane, the basilar membrane, that worms through the gyrations of the cochlear snail. Running parallel to the basilar membrane is the organ of Corti. The organ of Corti is where, finally, the message of the string—after many transformations—gets translated into nervous impulses. The details of all these transformations are complex, and fascinating to experts, but the big picture is simple and does not depend on those details. The big picture is that the frequency of the original vibration gets translated into firings of neurons that have the same frequency.One important aspect of the translation is especially pretty, and Pythagorean in spirit. It led Georg von Békésy to a Nobel Prize in 1961. Because the basilar membrane tapers along its length, different parts of it prefer to oscillate at different rates. The thicker parts have more inertia, so they prefer to vibrate more slowly, at lower frequencies, whereas the thinner parts prefer to vibrate at higher frequencies. (This effect is responsible for the difference in the overall pitch between typical male and female voices. At puberty the male vocal cords thicken markedly, leading to lower frequencies of vibration and a deepened voice.) Thus when a sound, after its many tribulations, sets the surrounding fluid into motion, the response of the basilar membrane will be different at different places along its length. A low-frequency tone will put the thicker parts into vigorous motion, while a high-frequency tone will put the thinner parts into vigorous motion. In this way, information about frequency gets encoded into information about position!If the cochlea is the eye of audition, the organ of Corti is its retina. The organ of Corti runs parallel to the basilar membrane, and close by. Its structure is complex in detail, but roughly speaking it consists of hair cells and neurons, one hair cell per neuron. The motion of the basilar membrane, coupled through intermediate fluid, exerts forces on the hair cells. The hair cells move in response, and their motion triggers electrical firing of the corresponding neurons. The frequency of the firing is the same as the frequency of stimulation, which in turn is the same as the frequency of the original tone. (For experts: The firing patterns are noisy, but they contain a strong component at the signal frequency.)Because the organ of Corti abuts the basilar membrane, its neurons inherit the position-dependent frequency response of that membrane. This is very important for our perception of chords, because it means that when several tones sound simultaneously, their signals do not get completely scrambled. Different neurons respond preferentially to different tones! This is the physiological mechanism that allows us to do such a good job of discriminating different tones.In other words, our inner ears follow the advice of Newton—and anticipate his analysis of light—by performing an excellent Analysis of the incoming sound into pure tones. (As we'll discuss later, our sensory ability to analyze the frequencies of signals in light, or in other words the color content of light, is based on different principles, and is much poorer.)This sets the scene for the third stage of our story. In it, signals from the primary sensory neurons in the organ of Corti are combined and passed on to subsequent neural layers in the brain. Here our knowledge is considerably less precise. But it is only here that we can finally come to grips with our main question:*Why* do tones whose frequencies are in ratios of small whole numbers sound good together?Let us consider what the brain is offered when two different sound frequencies play simultaneously. Then we have two sets of primary neurons responding strongly, each firing with the same frequency as the vibrations of the string that excites them. Those primary neurons fire their signals brainward, to "higher" levels of neurons, where their signals are combined and integrated.Some of the neurons at the next level will receive inputs from both sets of firing primaries. If the frequencies of the primaries are in a ratio of small whole numbers, then their signals will be synchronized. (For this discussion, we will simplify the actual response, ignoring the noise and treating it as accurately periodic.) For example, if the tones form an octave, one set will be firing twice as fast as the other, and every firing of the slower one will have the same predictable relationship to the firing of the former. Thus the neurons sensitive to both will then get a repetitive pattern that is predictable and easy to interpret. From previous experience, or perhaps by inborn instinct, those secondary neurons—or the later neurons that interpret their behavior—will "understand" the signal. For it will be possible to anticipate future input (i.e., more repetitions) in a simple way, and simple predictions for future behavior will be borne out, over many vibrations, until the sound changes its character.Note that the sound vibrations we can hear have frequencies ranging from a few tens to several thousand per second, so even brief sounds will produce many repetitions, except at the very low-frequency end. And at the low-frequency end our sense of harmony peters out, consistent with the line of thought we are pursuing.Higher levels of neurons, which combine the combiners, need coherent input to get on with their job. So if our combiners are producing sensible messages, and in particular if their predictions satisfy the test of time, it is in the interest of the higher levels to reward them with some kind of positive feedback, or at least to leave them in peace. On the other hand, if the combiners are producing wrong predictions, the mistakes will propagate up to higher levels, ultimately producing discomfort and a desire to make it stop.When will the combiners produce wrong predictions? That will happen when the primary signals are almost, but not quite, in synch. For then the vibrations will reinforce each other for a few cycles, and the combiners will extrapolate that pattern. They expect it to continue—but it doesn't! And indeed it is tones that are just slightly off—like C and C#, for example—that sound most painful when played together.If this idea is right, then the basis of harmony is successful prediction in the early stages of perception. (This process of prediction need not, and usually does not, involve conscious attention.) Such success is experienced as pleasure, or beauty. Conversely, unsuccessful prediction is a source of pain, or ugliness. A corollary is that by expanding our experience, and learning, we can come to hear harmonies that were previously hidden to us, and to remove sources of pain.Historically, in Western music, the palette of acceptable tone combinations has expanded over time. Individuals can also learn, by exposure, to enjoy tone combinations that at first seem unpleasant. Indeed, if we are built to enjoy *learning* to make successful predictions, then predictions that come too easily will not yield the greatest possible pleasure, which should also bring in novelty.PLATO I: STRUCTURE FROM SYMMETRY—PLATONIC SOLIDSThe Platonic solids carry an air of magic about them. They have been, and are, literally, objects to conjure with. They reach back deep into human prehistory, and live on as the generators of good or bad luck in some of the most elaborate of games, notably Dungeons & Dragons. Their mystique has inspired, besides, some of the most fruitful episodes in the development of mathematics and science. A worthy meditation on embodied beauty must dwell upon them.Albrecht Dürer, in his *Melancholia I* (figure 4), alludes to the allure of regular solids, although the solid that appears is not quite a Platonic solid. (Technically, it is a truncated triangular trapezohedron. It can be constructed by stretching out the sides of an octahedron in a peculiar way.) Perhaps the philosopher is melancholy because she can't fathom why a baleful bat dropped that particular, not quite Platonic, solid into her study, rather than a straightforward example.FIGURE 4. DÜRER'S MELANCHOLIA I. IT FEATURES A TRUNCATED PLATONIC SOLID, A VERY MAGIC SQUARE, AND MANY OTHER ESOTERIC SYMBOLS. TO ME, IT WELL DEPICTS THE FRUSTRATIONS I OFTEN ENCOUNTER WHEN USING PURE THOUGHT TO COMPREHEND REALITY. FORTUNATELY, IT'S NOT ALWAYS THIS WAY.Regular PolygonsTo appreciate the Platonic solids, let us start with something simpler: their closest two-dimensional analogue, regular polygons. A regular polygon is a planar figure with all equal sides that meet at all equal angles. The simplest regular polygon, with three sides, is an equilateral triangle. Next we have squares, with four sides. Then there are regular pentagons (the chosen symbol of the Pythagoreans, and also the design of a famous military headquarters), hexagons (the unit of a bee's hive and, as we shall see, of graphene), heptagons (various coins), octagons (stop signs), nonagons. . . . The series continues indefinitely: For each whole number, starting with three, there is a unique regular polygon. In each case, the number of vertices equals the number of sides. We can also consider the circle as a limiting case of regular polygon, where the number of sides becomes infinite.The regular polygons capture, in some intuitive sense, the notion of ideal regularity for planar "atoms." They will serve us as conceptual atoms, from which we build up richer and more complex ideas of order and symmetry.THE PLATONIC SOLIDSAs we move from planar to solid figures, searching for maximal regularity, we can generalize the regular polygons in various ways. A very natural choice, which turns out to be most fruitful, leads to the Platonic solids. We ask for solid bodies whose faces are regular polygons, all identical, that meet in identical fashion at every vertex. Then, instead of an infinite series of solutions, we find there are exactly five!FIGURE 5. THE FIVE PLATONIC SOLIDS: OBJECTS TO CONJURE WITH.These five Platonic solids are:

- The *tetrahedron*, with four triangular faces, four vertices, and three faces coming together at each vertex
- The *octahedron,* with eight triangular faces, six vertices, and four faces coming together at each vertex
- The *icosahedron,* with twenty triangular faces, twelve vertices, and five faces coming together at each vertex
- The *dodecahedron,* with twelve pentagonal faces, twenty vertices, and three faces coming together at each vertex
- The *cube*, with six square faces, eight vertices, and three faces coming together at each vertex

The existence of those five solids is easy to grasp, as one can imagine and construct models without great difficulty. But why are there just those five? (Or are there others?)To get our head around that question, we notice that the vertices of the tetrahedron, octahedron, and icosahedron feature three, four, and five triangles coming together, and ask, What happens if we continue to six? Then we realize that six equilateral triangles sharing a common vertex *lie flat.* Repeating that flat building block will not allow us to complete a finite figure, bounding a solid volume. Instead, it leads to an infinite dissection of a plane, as shown in figure 6:Platonic ProdigalsFIGURE 6. THE THREE INFINITE PLATONIC SURFACES. ONLY FINITE PORTIONS ARE SHOWN HERE. THESE THREE REGULAR DISSECTIONS OF A PLANE CAN AND SHOULD BE CONSIDERED RELATIVES OF THE TRADITIONAL PLATONIC SOLIDS—THEIR PRODIGAL SIBLINGS THAT WANDER OFF AND NEVER RETURN.We find similar results if we put together four squares, or three hexagons. These three regular dissections of a plane are worthy supplements to the Platonic solids. We will find them embodied in the microcosm (figure 29).If we try to put together more than six equilateral triangles, four squares, or three of any of the larger regular polygons, we run out of room—we simply can't accommodate the accumulated angles. And so the five Platonic solids are the only finite regular solids.It is remarkable that a specific finite number—that is, five—emerges from considerations of geometric regularity and symmetry. Regularity and symmetry are natural and beautiful things to consider, but they have no obvious or direct connection to specific numbers. Plato interpreted this profound emergence in an astonishingly creative way, as we shall see.PrehistoryFamous people often get credit for the discoveries of others. This is the “Matthew Effect” identified by the sociologist Robert Merton, based on this observation from the Gospel of Matthew:For unto every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken even that which he hath.So it is for the Platonic solids.At the Ashmolean Museum of Oxford University you can see a display of five carved stones dating from 2000 BCE Scotland that appear to be realizations of the Platonic solids (though some scholars dispute this). They were most likely used in some sort of dice game. Let us imagine cave people huddled around the communal fire, rapt in paleolithic Dungeons & Dragons. But it was probably Plato’s contemporary Theaetetus (417–369 BCE) who first *proved* mathematically that those five bodies are the only possible regular solids. It’s not clear to what extent Theaetetus was inspired by Plato, or vice versa, or whether it was something in the Athenian air they both breathed. In any case, the Platonic solids got their name because Plato used them creatively, in work of imaginative genius, to construct a visionary theory of the physical world.FIGURE 7. PRE-PLATONIC ANTICIPATIONS OF THE PLATONIC SOLIDS, PROBABLY USED IN DICE GAMES CIRCA 2000 BCE.Going back much further, we now realize that some of the biosphere’s simplest creatures, including viruses and diatoms (not pairs of atoms, but marine algae that often grow elaborate Platonic exoskeletons), not only “discovered” but have literally embodied the Platonic solids since long before humans walked the Earth. The herpesvirus, the virus that causes hepatitis B, the HIV virus, and many other nasties are shaped like icosahedra or dodecahedra. They encase their genetic material—either DNA or RNA—in protein exoskeletons, which determine their external form, as seen in plate D. The exoskeleton is color-coded in such a way that identical colors indicate identical building blocks. The dodecahedron’s signature triply meeting pentagons leap to the eye. If we join the centers of the blue regions with straight lines, an icosahedron emerges.More complex microscopic creatures, including the radiolaria lovingly portrayed by Ernst Haeckel in his marvelous book *Art Forms in Nature,* also embody the Platonic solids. In figure 8 it is the intricate silica exoskeletons of these single-cell organisms that we see. The radiolarians are an ancient life-form, represented in the earliest fossils. They continue to thrive in the oceans today. Each of the five Platonic solids is realized in a number of species. Several species names enshrine those shapes, including *Circoporus octahedrus, Circogonia icosahedra,* and *Circorrhegma dodecahedra.*Euclid’s InspirationEuclid’s *Elements* is, by a wide margin, the greatest textbook of all time. It brought system and rigor to geometry. From a larger perspective it established, by example, the method of Analysis and Synthesis in the domain of ideas.Analysis and Synthesis is Isaac Newton’s, and our, preferred formulation of “reductionism.” Here is Newton:FIGURE 8. RADIOLARIA BECOME VISIBLE UNDER A MODEST MICROSCOPE. THEIR EXOSKELETONS OFTEN EXHIBIT THE SYMMETRY OF PLATONIC SOLIDS.By this way of Analysis we may proceed from Compounds to Ingredients, and from Motions to the Forces producing them; and in general, from Effects to their Causes, and from particular Causes to more general ones, till the Argument end in the most general. This is the Method of Analysis: And the Synthesis consists in assuming the Causes discover’d, and establish’d as Principles, and by them explaining the Phænomena proceeding from them, and proving the Explanations.This strategy parallels Euclid’s approach to geometry, where he proceeds from simple, intuitive *axioms* to deduce rich and surprising consequences. Newton’s great *Principia,* the founding document of modern mathematical physics, also follows Euclid’s expository style, building from axioms to major results step-by-step through logical construction.It is important to emphasize that axioms (or laws of physics) don’t tell you what to do with them. By stringing them together without purpose, it’s easy to generate hosts of forgettable, worthless truths—like a play or a piece of music that wanders aimlessly, arriving nowhere. As those who have attempted to deploy artificial intelligence to do creative mathematics have discovered, identifying *goals* is often the hardest challenge. With a worthy goal in mind, it becomes easier to find the means to achieve it. My all-time favorite fortune cookie summed this up brilliantly:The work will teach you how to do it.Also, of course, as a matter of presentation, it’s attractive to students and potential readers to have an inspiring goal in sight—and impressive for them to realize, at the start, that they can look forward to experiencing an amazing feat of construction that builds, by inexorable steps, from “obvious” axioms to far-from-obvious conclusions.So:

What was Euclid’s goal in the *Elements*? The thirteenth and final volume of that masterpiece concludes with constructions of the five Platonic solids, and a proof that there are only five. I find it pleasant—and convincing—to think that Euclid had this conclusion in mind when he began drafting the whole, and worked toward it. In any case, it is a fitting, fulfilling conclusion.Platonic Solids as AtomsThe ancient Greeks recognized four building blocks, or elements, for the material world: fire, water, earth, and air. You might notice that four, the number of elements, is close to five, the number of regular solids. Plato certainly did! One finds, in his influential, visionary, inscrutable *Timaeus*, a theory of the elements based on the solids. Here it comes:Each of the elements is built from a different variety of atom. The atoms take the form of Platonic solids. The atoms of fire are tetrahedra, the atoms of water are icosahedra, the atoms of earth are cubes, and the atoms of air are octahedra.There is a certain plausibility to these assignments. They have explanatory power. The atoms of fire have sharp points, which explains why contact with fire is painful. The atoms of water are most smooth and well-rounded, so they can flow around one another smoothly. The atoms of earth can pack closely, and fill space without gaps. Air, being both hot and wet, features atoms intermediate between those of fire and water.Now while five is close to four, it is not quite equal to it, so there cannot be a perfect match between regular solids, regarded as atoms, and elements. A merely brilliant thinker might have been discouraged by that difficulty, but Plato, a genius, was undaunted. He took it as a challenge and an opportunity. The remaining regular solid, the dodecahedron, he proposed, does figure in the Creator’s construction, but not as an atom. No, the dodecahedron is no mere atom—rather, it is the shape of the Universe as a whole.Aristotle, who was forever determined to one-up Plato, put forward a different, more conservative and intellectually consistent variation of that theory. Two of that influential philosopher’s big ideas were: that the Moon, planets, and stars inhabit a celestial realm made from stuff different from what we find in the mundane world; and that “Nature abhors a vacuum,” so that the celestial spaces could not be empty. Thus consistency required there to be a fifth element, or quintessence, different from earth, air, fire, and water, to fill the celestial realm. Dodecahedra, then, find their place as the atoms of quintessence, or ether.It is difficult to agree, today, with the details of these theories, in either version. We haven’t found it useful, in science, to analyze the world in terms of those four (or five) elements. Nor are modern atoms hard, solid bodies, much less realizations of the Platonic solids. Plato’s theory of the elements, seen from today’s perspective, is both crude and, in detail, hopelessly misguided.Structure from SymmetryAnd yet, though it fails as a scientific theory, Plato’s vision succeeds as prophecy and, I would claim, as a work of intellectual art. To appreciate those larger virtues, we have to step away from the details, and look at the bigger picture. The deepest, core intuition of Plato’s vision of the physical world is that the physical world must, fundamentally, embody beautiful concepts. And this beauty must be of a very special kind: the beauty of mathematical regularity, of perfect symmetry. For Plato, as for Pythagoras, that intuition was at the same time a faith, a yearning, and a guiding principle. They sought to harmonize Mind with Matter by showing that Matter is built from the purest products of Mind.Revue de presse
“Mr. Wilczek takes the reader on an expertly curated tour across 2,500 years of philosophy and physics…One of the great pleasures of Mr. Wilczek’s book is his wide-ranging interest in the way the beauty he finds in symmetry appears across human experience. …He has accomplished a rare feat: Writing a book of profound humanity based on questions aimed directly at the eternal.”—*The Wall Street Journal*

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“*A Beautiful Question* is a compelling introduction to the triumphs and challenges of modern physics, presented as a meditation on the role of aesthetics in the search for a deeper understanding of nature, and the deeper meanings of that search for humanity. Full of historical background and infused with the author’s generous humanity, this is indeed a beautiful book, one I recommend to anyone interested in where science is going, written by someone who, by his many lasting contributions to science, has earned our attention.”—Lee Smolin, author of *Time Reborn* and *The Trouble with Physics*

“In this exquisite and remarkably accessible book, Frank Wilczek explores our cosmos as a work of art, revealing hidden beauty at all levels from the Galactic realm down to the subatomic microworld that his trailblazing research has elucidated. His ability to see what others overlook makes him an inspiring guide not only for scientists, but also for artists and all curious people.”—Max Tegmark, author of *Our Mathematical Universe*

“If you’ve ever wondered what physicists mean when they describe a theory as ‘beautiful,’ *A Beautiful Question* is the ideal place to find out. Wilczek is both one of the greats of the subject, and not afraid to engage non-technically with the wonderful complexities and intangibilities of the mysterious beauty that lies at the core of our understanding of the physical world.”—Peter Woit, author of *Not Even Wrong*

“Anyone who has studied physics knows the startling beauty of those rare times when the clouds part and you see that math and reality are the same thing. With Wilczek’s new book, readers can catch a glimpse of that beauty without having to know the math.”—Noah Smith, Stony Brook University; author of *Noahpinion*

“In contemporary art, Beauty has faded, a prosaic artifice, a distraction from deeper raw truths, maybe even ugly truths. To the exceptional physicist Frank Wilczek, Beauty has proven a luminous ally, a faithful advisor in his discoveries of remarkable truths about the world. Ever in pursuit of truth, Frank guides us in a calm and winsome meditation on this subtle question: Is the world beautiful?”—Janna Levin, author of *How the Universe Got Its Spots*

A beautiful treatise on a beautiful universe, this delightful series of meditations on the nature of beauty and the physical universe roams from music, to color vision, to fundamental ideas at the very forefront of physics today. In lesser hands such a romp could easily degenerate into a kind of new age mystical mumbo jumbo. However, Frank Wilczek is one of the deepest, most creative, and most knowledgeable theoretical physicists alive today. Read him or listen to him and you will never think about the universe the same way again. And if your experience is like mine over the years, you will definitely be the better for it."—Lawrence Krauss, author of *A Universe from Nothing* and *The Physics of Star-Trek*

"Frank Wilczek starts this fascinating book with the intriguing question: Does the world embody beautiful ideas? What follows is a masterful, intellectual journey, surveying a breathtaking tapestry of physics, art, and philosophy. One could ask Wilczek's question differently: Does this book embody beautiful ideas? The answer would be a resounding Yes!"—Mario Livio, astrophysicist, author of *Brilliant Blunders*

"Before there was Science, there was Natural Philosophy. In this authoritative, ever-surprising, and lavishly illustrated account, Frank Wilczek brings the grand quest that so captivated Pythagoras, Copernicus, Galileo, Newton, Maxwell, Einstein, Noether, and a host of others both up to date and back to life."—George Dyson, author of *Turing's Cathedral*

"A truly beautiful book, in design, in content, in the insights that Frank Wilczek shares. This book helps me see how one of the world's leading thinkers thinks, using beauty as a tool, as a guide in finding not only the right problems but the right solutions. In Wilczek's mind, there is no clear separation between physics, art, poetry, and music. Why do physicists call their theories beautiful? Immerse yourself in this book, wallow in it, sit back and relax as you wander through it, and you'll soon understand."—Richard Muller, author of *Physics for Future Presidents*

"For a century, science has invalidated 'soft' questions about truth, beauty, and transcendence. It took considerable courage therefore for Frank Wilczek to declare that such questions are within the framework of 'hard' science. Anyone who wants to see how science and transcendence can be compatible must read this book. Wilczek has caught the winds of change, and his thinking breaks through some sacred boundaries with curiosity, insight, and intellectual power."—Deepak Chopra, M.D. Présentation de l'éditeur
Does the universe embody beautiful ideas?

Artists as well as scientists throughout human history have pondered this "beautiful question." With Nobel laureate Frank Wilczek as your guide, embark on a voyage of related discoveries, from Plato and Pythagoras up to the present. Wilczek's groundbreaking work in quantum physics was inspired by his intuition to look for a deeper order of beauty in nature. In fact, every major advance in his career came from this intuition: to assume that the universe embodies beautiful forms, forms whose hallmarks are symmetry—harmony, balance, proportion—and economy. There are other meanings of "beauty," but this is the deep logic of the universe—and it is no accident that it is also at the heart of what we find aesthetically pleasing and inspiring.

Wilczek is hardly alone among great scientists in charting his course using beauty as his compass. As he reveals in *A Beautiful Question*, this has been the heart of scientific pursuit from Pythagoras, the ancient Greek who was the first to argue that "all things are number," to Galileo, Newton, Maxwell, Einstein, and into the deep waters of twentiethcentury physics. Though the ancients weren't right about everything, their ardent belief in the music of the spheres has proved true down to the quantum level. Indeed, Wilczek explores just how intertwined our ideas about beauty and art are with our scientific understanding of the cosmos.

Wilczek brings us right to the edge of knowledge today, where the core insights of even the craziest quantum ideas apply principles we all understand. The equations for atoms and light are almost literally the same equations that govern musical instruments and sound; the subatomic particles that are responsible for most of our mass are determined by simple geometric symmetries. The universe itself, suggests Wilczek, seems to want to embody beautiful and elegant forms. Perhaps this force is the pure elegance of numbers, perhaps the work of a higher being, or somewhere between. Either way, we don’t depart from the infinite and infinitesimal after all; we’re profoundly connected to them, and we connect them. When we find that our sense of beauty is realized in the physical world, we are discovering something about the world, but also something about ourselves.

Gorgeously illustrated, *A Beautiful Question* is a mind-shifting book that braids the age-old quest for beauty and the age-old quest for truth into a thrilling synthesis. It is a dazzling and important work from one of our best thinkers, whose humor and infectious sense of wonder animate every page. Yes: The world is a work of art, and its deepest truths are ones we already feel, as if they were somehow written in our souls.
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